Worksheet lecture 21, Neyman-Pearson Lemma. Math 481

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1. Recall $Exp(\lambda)$ population has PDF : $f(x) = \frac{1}{\lambda}e^{-\frac{x}{\lambda}}, x > 0$.
The null hypothesis $\lambda = 10$ is to be tested against the alternative $\lambda = 5$ using observed sample data $x_1, x_2 \ldots x_n$. Use the Neyman Pearson Lemma, to obtain a test with the most power when the size of the critical region is to be a fixed α.

$L_0 = f(x_1|\lambda=10)\, f(x_2|\lambda=10) \ldots = \frac{1}{10^n} e^{-\frac{1}{10}(\Sigma x)} = \frac{1}{10^n} e^{-\frac{n\bar{x}}{10}}$

$L_1 = \frac{1}{5^n} e^{-\frac{n\bar{x}}{5}}$

$\frac{L_0}{L_1} = \frac{5^n}{10^n}\left(e^{\frac{n\bar{x}}{5}} e^{-\frac{n\bar{x}}{10}}\right) = \left(\frac{1}{2}\right)^n e^{\frac{n\bar{x}}{10}}$

$\frac{L_0}{L_1} \leq k$

$\left(\frac{1}{2}\right)^n e^{\frac{n\bar{x}}{10}} \leq k$

$e^{\frac{n\bar{x}}{10}} \leq k$

$\frac{n\bar{x}}{10} \leq k$

$\bar{x} \leq k$

2. In the previous question suppose $n = 1$ (sample size of one) and the probability of type 1 error $\alpha = 0.05$. What is the critical region in this case?

$n=1$, thus $\bar{x} = x$

$\lambda = 10$ $\int_0^y \frac{1}{\lambda} e^{-x/\lambda}\,dx = 1 - e^{-y/\lambda} = F(y)$

$P(x \leq k) = \alpha$ $F(k) = \alpha = 1 - e^{-k/10}$ $1-\alpha = e^{-k/10}$

$\alpha = .05$ $\ln(1-\alpha) = -k/10$

$k = -10\ln(1-\alpha) = -10\ln(.95)$

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